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BALLET BALANCE STRATEGIES

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Abstract

Animating physically realistic human characters is challenging, since human observers are highly tuned to recognize human cues such as emotion and gender from motion patterns. Any new effort towards improving the physical realism of animating the human body is therefore valuable both for application and research purposes.

The main contribution of this paper is a new model firmly based on biomechanics. The new model has been developed to animate some basic steps of ballet dancers, and it is supported by computer simulated experiments showing good agreement with biomechanical measurements of real-life dancers.

Keywords: Biomechanics, Balance Strategy, Weight shift strategy, Control Mechanism.

Nomenclature

The following symbols are used in this paper:

τ	Joint torque
$\theta, \Delta\theta$	Joint angle and angular update
k_s, k_d	Spring constancts
$\vec{r}_i$	Position of body part *i*
$\vec{r}_{cm}, \vec{r}_{cp}$	Position of center of mass and pressure
m_i, M	Mass of body part *i* and total mass
p_j	Contact point
n_j	Normal force at p_j
$\vec{f}_j$	Contact force at p_j

Introduction

A long term goal of computer graphics is to increase realism and believability into computer generated animations and pictures [1, 31, 18, 5, 37, 4]. With improved rendering techniques, the lack of physical realism and believability is becoming increasingly obvious and annoying to the common observer, and one accompanying long term goal in animation is to increase physical realism by using physics to model plausible behavior and movement of computer models. This known as physics-based animation.

Physics-based animation is a highly interdisciplinary field, based on engineering, physics, and mathematics [42, 36, 3]. Most noticeable is simulation models based on traditional engineering methods used in robotics and construction [9, 14], where forward dynamics is the most popular technique.

In an animated movie it is generally believed that relying 100% on physical principles inhibits the creativity and aesthetics of an animator, and animators typically work using the “Principles of Animation” [25, 15] such as follow through and exaggeration of the motion, to convey the emotions of a character. The implication is that characters may be put in unnatural poses leading to penetrations and cloth tangling.

In recent years the emphasis on physics-based animation have given rise to a new field, “plausible simulation” [6], where new techniques have been proposed, such as sampling the entire range of possible simulations using forward dynamics [19] and optimization of physical constraints [45, 39, 38, 12]. However, animating physically realistic human characters has proven to be very challenging, since human observers are fine tuned to recognize human cues from motion patterns. This is in contrast to the mathematical complexity of simulating natural turbulent phenomena like water and smoke.

This paper studies biomechanical and ballet inspired

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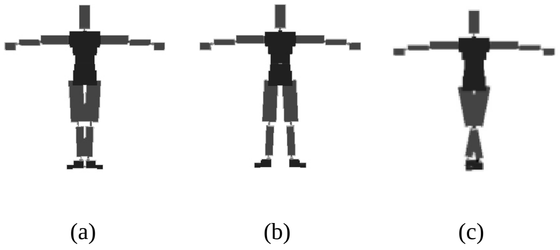

(a) (b) (c)

Figure 1: Three ballet poses: (a) First, (b) Second, and (c) Fifth. The feet are positioned in the frontal planed in all poses.

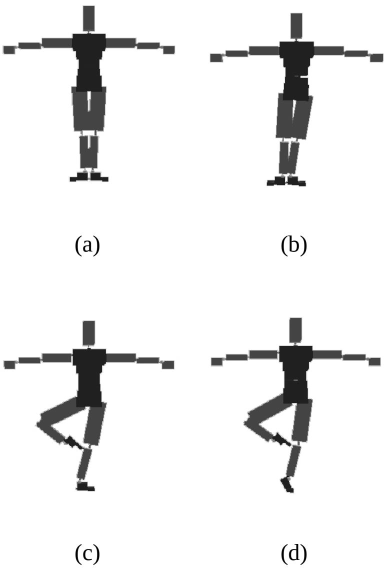
balance and weight shifting strategies. Ballet is a balance art and is a prime focus for learning about human balance and weight shifting strategies. This paper is based on the master thesis by N.N. [35], where an articulated figure is modeled to perform some basic balances and movements of a ballet dancers. N.N. has combined a bachelor in music and dance with ballet dancing and a master degree in computer science.

The articulated figure is initiated from one of the standard poses in quiet standing shown in Figure 1, and the weight shifting strategies used to obtain a quiet standing on one toe will be described. The four sub-goals are shown in Figure 2: Balancing on two legs, weight shift to the supporting leg, balancing on one leg, and balancing on the toe of one leg. The new model is firmly based on biomechanics and is supported by computer simulated experiments showing good agreement with biomechanical measurements of real-life dancers.

Survey of past work

The study of balance has been performed in at least three seperate areas of research: Biomechanics, Robotics, and Animation. We will in the following highligt some research from these areas.

Biomechanics and the study of ballet: Ballet is an art-form, where balance plays a central role. Classical ballet techniques are throughly described in the literature, see e.g. [17, 43]. Biomechanical studies of ballet has mainly been studied through injury cases, e.g. [13]. The most common biomechanical investigations into balancing of the human body


(a) (b)

(c) (d)

Figure 2: Shifting the weight from both legs in first pose (a) to the left leg (b), to a one-legged stand (c), and finally to a one-toed (d) quiet standing.

is through the inverted pendulum model [28]. Empirical investigations have shown that the velocity of center of mass plays a role in balancing [44]. Empirical studies on real humans have been performed on balancing of humans versus the position of the center of mass [34]. Weight shifts have been measured using a force-platform [32, 33]. Finally, a thorough measurement of the properties of mass and inertia of human body parts may be found in [10].

Robotics: An early work on balancing robots may be found in [41]. Weight shift strategies for walking are often performed through dynamic walking machines, where there is more or less degree of control involved in the walking cycle [29, 24, 40]. Recently, a number of spectacular and stable humanoid robots have been produced starting with the Honda robot [22]. Especially in relation to our work, the center of pressure has recently been introduced as a stable control mechanism for balancing robots [21, 20].

Animation: A very early mentioning of simulated human motion is given in [2], and early implementations may be found in [16, 30]. The implementation of dancing models is only scarcely discussed in the literature, however one exception is [8]. A major in-

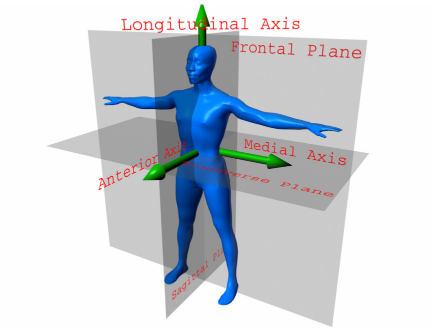


Figure 3: Sagittal, Transverse, and Frontal planes and Medial, Longitudinal and Anterior axis.

spiration for our work has been [26, 46], where center of mass is used to control the balance and motion of a humanoid model of varying complexity. Alternative to center of mass strategies are inverse kinematics e.g. [27], energy models e.g. [7], and learning approaches e.g. [11].

The Biomechanics of Quiet Standing

A human in quiet standing may be modeled by an articulated figure [26, 46] consisting of a set joints and a set of links representing body parts. The set of possible joints consist of revolute (1 Degree of Freedom (DOF)), universal (2 DOF), and ball-and-socket (3 DOF) joints. In this work, we have used Wooten's model [46], which contains 28 DOFs, and uses real measurements of the mass, m_i, center of mass, $\vec{r}_i$, and moments of inertial of all the body parts [10]. The ankle and hip joints is of particular importance for this paper, and they are modeled by an universal joint and a ball-and-socket joint respectively.

Traditionally in biomechanics and anatomy, motion and orientation are described in three planes: The Sagittal (x-axis), the Transverse (y-axis) and the Frontal (z-axis) plane. These planes are illustrated in Figure 3. Measurements of angles and positions are traditionally also performed in these planes [32] by projection onto the respective planes and axes. Typical projections are: The position of the center of mass, the position of the center of pressure, angle of joints, and the direction of gravity. Although it seems needlessly complicated to work with the projections rather than the underlying 3D geometry, it allows for comparison with the substantial biomechanical literature.

Muscles are used to move and sustain posture of the human skeleton, and our articulated figure is supplemented by an actuator system, which applies joint torques according to a simple damped angular spring model,

$$\tau = k_s^{\text{muscle}}\left(\theta^{\text{target}} - \theta^{\text{current}}\right) - k_d^{\text{muscle}}\dot{\theta}^{\text{current}}. \qquad (1)$$

In the equation, τ is the length of the torque vector, θ^{target} and θ^{current} are target and current angles, $\dot{\theta}^{\text{current}}$ is the current velocity of the angle, and k_s^{muscle} and k_d^{muscle} are spring and damping constants.

A balance control strategy is a function that determine updates, $\Delta\theta$'s, based on the current state of the articulated figure, θ^{current}'s, such that the model will converge towards a desired state of quiet balance. I.e. the strategy iteratively determines new parameter values, θ^{new}'s as,

$$\theta^{\text{new}} = \theta^{\text{current}} + \Delta\theta. \qquad (2)$$

In the rest of this article, it will be assumed that the model is placed on a planar floor, and the contact between the floor and the feet is represented by a set of coplanar contact points, $\vec{p}_j$. The support polygon is defined as the 2D convex hull of all the contact points. The center of mass is of the human model is defined as

$$\vec{r}_{\text{cm}} = \frac{1}{M}\sum_i^N m_i\vec{r}_i, \qquad (3)$$

where $M = \sum_i^N m_i$ is the total mass, N is the number of body parts in the model, m_i is the individual weights of the body parts, and r_i are their locations. Note that center of mass is not fixed w.r.t. any location of the body during motion of the individual body parts. The center of pressure is defined as

$$\vec{r}_{\text{cp}} = \frac{1}{||\vec{n}||}\sum_j^K ||\vec{n}_j||\vec{p}_j, \qquad (4)$$

where $\vec{n} = \sum_j^K \vec{n}_j$ is the total normal force acting on the human model, $\vec{n}_j$ is the normal force applied to the human model at the j'th contact point $\vec{p}_j$, and K is the number of contact points.

For simplicity, a spring model for the floor contact forces is used, where the contact force of the j'th contact is

$$\vec{f}_j = k_s^{\text{contact}}\left(\vec{p}_j^{\,\text{initial}} - \vec{p}_j\right) - k_d^{\text{contact}}\dot{\vec{p}}_j. \qquad (5)$$

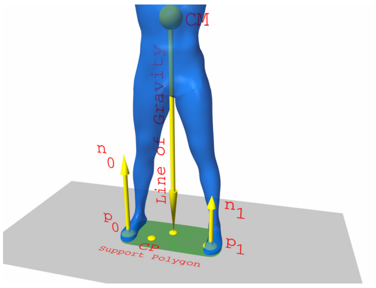


Figure 4: Illustration of biomechanical definitions: Center of mass, center of pressure, support polygon and line of gravity

In the equation, $\vec{p}_j^{\,\text{initial}}$ is the initial point of contact, $\vec{p}_j$ is the current contact point, $\dot{\vec{p}}_j$ is the velocity of the current contact point, and k_s^{contact} and k_d^{contact} are spring and damping constants. The vector, $\vec{n}_j$, is calculated as the projection of $\vec{f}_j$ onto the contact normal of the floor, and the tangential part is a simple model of frictional force.

Contact forces can only be repulsive, and attractive contact forces, f_j, are therefore set to zero. Slipping is obtained by setting $\vec{p}_j^{\,\text{initial}}$ equal to $\vec{p}_j$, when the magnitude of the tangential force component exceed a multiple of the magnitude of the normal force component, $||\vec{f}_{\text{friction}}|| \leq -\mu||\vec{n}||$.

The line of gravity is defined as the line going from the center of mass to the ground in the direction of the gravitational field. The point of intersection between the line of gravity and the floor is referred to as the projection of the center of mass. These concepts are illustrated in Figure 4.

Balance is defined as an objects ability to maintain quiet standing, where quiet standing is the state, where the projection of the center of mass is kept within the support polygon [23, 26, 46]. The implication is that the greater support polygon, the lower center of mass, the more stable the balance and vice versa. The human body has a highly placed center of mass over a rather small support polygon, and as such the human body behaves as an inverted pendulum.

In the remainder of this paper, control strategies for maintaining quiet standing and moving an articulated figure from one pose of quiet standing to another will be discussed.

Balance Strategies: Mass Center versus Pressure Center

In the following, two strategies for balance will be compared: The center of mass strategy and the center of pressure strategy. The analysis are done on an articulated figure based on [46] standing with parallel feet. The balance is controlled by the ankle and hip joint and only in Sagittal plane in agreement with [44].

The center of mass strategy is the traditional balance strategy, where the angular change is controlled as a function of the projection of the center of mass onto the support plane [26, 46]:

$$\Delta\theta = k_s^{\text{cm}}\left(r_{\text{cm}}^{\text{current}} - r_{\text{cm}}^{\text{target}}\right) - k_d^{\text{cm}} v_{\text{cm}}, \tag{6}$$

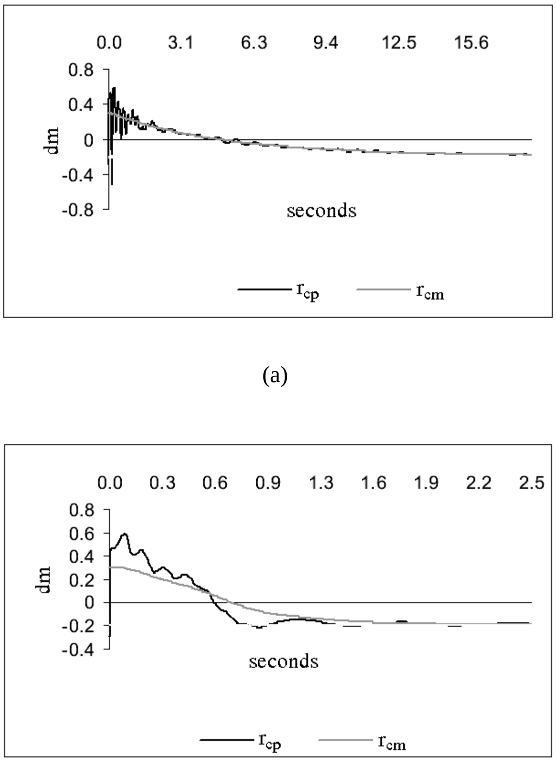
where $\Delta\theta$ is the angular change of the ankle in the Sagittal plane, $r_{\text{cm}}^{\text{current}}$ and $r_{\text{cm}}^{\text{target}}$ are the projections of the current and target positions of the center of mass onto the anterior axis, v_{cm}, is the velocity vector of the center of mass projected onto the anterior axis, and k_s^{cm} and k_d^{cm} are control parameters.

In the center of pressure strategy, the model uses the center of pressure to control the center of mass, and when the positions are right above each other the pendulum is in perfect balance. The goal of this strategy is therefore to calculate a desired position of the center of pressure, and use this for controlling the muscles.

$$\begin{aligned}
\Delta r_{\text{cp}} &= k_s^{\text{cm}}\left(r_{\text{cm}}^{\text{current}} - r_{\text{cm}}^{\text{target}}\right) - k_d^{\text{cm}} v_{\text{cm}}, && (7)\\
r_{\text{cp}}^{\text{target}} &= \Delta r_{\text{cp}} + r_{\text{cm}}, && (8)\\
\Delta\theta &= k_s^{\text{cp}}\left(r_{\text{cp}}^{\text{current}} - r_{\text{cp}}^{\text{target}}\right) - k_d^{\text{cp}} v_{\text{cp}}, && (9)
\end{aligned}$$

where Δr_{cp} is the positional change of the projection of the center of pressure onto the anterior axis, $r_{\text{cp}}^{\text{current}}$ and $r_{\text{cp}}^{\text{target}}$ are the projections of the current and target positions of the center of projection onto the anterior axis, and k_s^{cp} and k_d^{cp} are control parameters.

In Figure 5 the dynamics of the projected center of mass and the center of pressure on the anterior axis is shown. The model manages to balance using both strategies, but the center of pressure strategy requires only approximately 1.5 sec., while the center of mass strategy requires almost 10 sec.. In addition, the center of pressure strategy also has the smallest amplitude of the oscillations of the center of pressure, which means that it has the best control over the contact with the ground.



(a)

(b)

Figure 5: Comparison of the dynamics of the ankle for the two strategies: (a) Center of mass strategy and (b) Center of pressure strategy. Motion is restricted to the Sagittal plane.

Dynamics of a Ballet Dancer

Quiet standing on the toe of one leg is central in all ballet training. This is demanding, since the dancer has to balance on a very small support polygon while at the same time looking at ease. Both in quiet balance and in most of the basic ballet exercises, the legs are strictly separated into the working leg (doing the exercise) and the supporting leg. Shifting the weight between the legs is therefore an important task, and it should preferably be done without drawing the attention of the audience. In [35], strategies for obtaining a balance on the left leg with the right foot by the left knee were developed as illustrated in Figure 2. This pose is used, when dancers turn in a pirouette, and it is perhaps the most basic of all balances in ballet training.

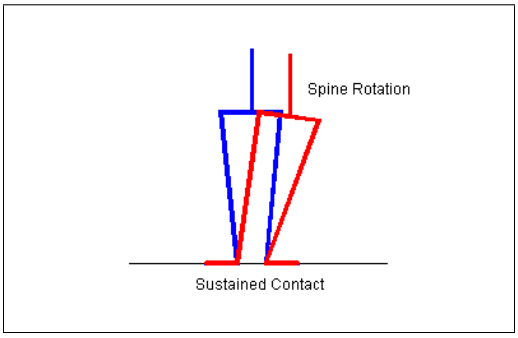


Figure 6: A rotation in the ankles with straight legs implies a rotation in the hip, and a rotation in the spine is required in order to keep the upper body vertical.

Weight Shifting Strategies

The weight shifting strategy described in the following was inspired by the analysis of real dancers presented in the literature [32], where it was shown that changes in angles in the hips and ankles are nearly identical during weight shift, and that the center of pressure start moving towards the working leg and end up being on the supporting leg. The movement of the center of pressure was tested on the articulated figure for the shifting of the weight in the frontal plane from a position between the feet to the left foot as shown in Figures 2(a)-2(b), and the center of pressure strategy showed good agreement with the measurements on real dancers.

To shift the weight, the left ankle was designated to be the controlling joint using (9), while the remaining joints were used to keep balance and to control the position of the upper body. Keeping the right foot flat on the ground, the angular changes in the right ankle and the hips are calculated from the angular change in the left ankle.

Ballet aesthetics requires that the upper body is kept parallel to the line of gravity during a weight shift in the frontal plane with both feet fixed on the floor. It has been claimed [32] that dancers keep their upper body vertical by a counter rotation in the hip joints, however this is only physically possible when the legs are parallel as illustrated in Figure 6. Ballet dancers compensate for the hip rotation by a counter rotation in the lumbar region of the lower back [17], and therefore a control function in the spine is required. Dancers control the body center by the stomach muscles, and experience [35] has shown that these stomach muscles are extremely important for aesthetic motion of the articulated figure, thus a control function in the pelvis is used to inhibit rotation in the Sagittal plane. Both control functions are modeled using a spring law similar to (1).

The final weight shift of the articulated figure is shown in Figure 2. The resulting articulated figure agrees with measurements performed on real ballet dances [32] as follows: The measured angles in quiet standing are identical, and at initial weight shift, the center of pressure starts to move towards the right foot and ends in a position on the left foot as seen in Figure 5(b).

Movement to Quiet Standing on One Toe

The final movements to obtain a one-toed quiet standing is achieved by lifting the non-supporting leg, and shifting from a foot stand to a toe stand. See Figure 2. It is not difficult to raise the leg, however the major challenge is to keep balance on a very small support polygon.

Two strategies have been developed: A strategy for raising the right leg, and a strategy for making a weight shift to the toe. Similarly to the weight shifting balance strategy, both are based on the center of pressure strategy.

To raise the leg, the left ankle is kept as the controlling joint, and the center of pressure strategy is used to keep the balance on the center of the left foot. To shift the weight to one toe, the center of pressure strategy is used in two steps: Firstly, for the controlling ankle joint, and secondly to control the toe joint, when the ankle has been straightened.

Lifting the right leg to an aesthetically pleasing pose is performed using a spring law. Both the lifting of the leg and the shifting of the weight to the toe produces a motion of the center of mass, and the left hip is used to maintain balance by (6). The spine and pelvis are controlled as explained in the previous section. The analysis of the results shows a motion of the center of pressure, which is not entirely in agreement with measurements on real dances [32].

Discussion

This paper has compared two strategies for an articulated figure in quiet standing: Center of mass and center of pressure. It has been demonstrated that controlling balance using the center of pressure is the fastest strategy and is also in best agreement with biomechanical measurements on ballet dancers. From a ballet point of view, controlling the center of pressure is a way of controlling the contact of the feet with the ground. Ballet-dancers are very much aware of the relation between their feet and the ground, since it strongly influences their balance, their stance, and the audience's impression of the dancer's body. A ballet dancer must have a light impression.

Further in contrast to some speculations in the literature, the model presented in this paper has shown that it is most natural to use the ankles and not the hip joints for balance control.

Using spring laws for modeling actuator forces and contact forces are attractive due to their simplicity although they require a lot of parameter tuning in practice.

Balance and weight shifting are the most basic techniques learnt in ballet. Future steps in our research will be to develop strategies for exercises on one leg. However, it remains to be demonstrated that a small collection of motion strategies are sufficient to simulate the physical motion of human beings. It is our belief that this is the case.

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